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Numerical Methods – Final Project

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# Problem 1

The following data are measured precisely:

A close up of numbers

Description automatically generated

1. Use Newton interpolating polynomials to determine z at t = 2.5. Make sure that you order your points to attain the most accurate results. What do your results tell you regarding the order of the polynomials used to generate the data?
2. Use a third-order Lagrange interpolating polynomial to determine y at 2.5.

## Using Newton Interpolating Polynomial

Given our points T and Z, we start computing the first order divided difference:

### Formula

We now construct the Divided Difference table using this formula, starting with the First Order.

For T = 2, to T = 2.1:

For T = 2.1 to T = 2.2:

For T = 2.2 to T = 2.7:

For T = 2.7 to T = 3:

For T = 3 to T = 3.4:

For the second order, we use the values gotten when finding the first order divided difference.

For T = 2 to T = 2.2:

For T = 2.1 to T = 2.7:

For T = 2.2 to T = 3:

For T = 2.7 to T = 3.4:

Now, we continue with the third order using the computer values from the second order.

For T = 2, T = 2.2 and T = 2.7 using deltas from second order: T = 2.1 and T = 2:

For T = 2.1, T = 2.7 and T = 3 using deltas from second order: T = 2.2 and T = 2.1:

For T = 2.2, T = 3 and T = 3.4 using deltas from second order: T = 2.7 and T = 2.2:

Finally, we move to the fourth order, we again use the values from the previous order.

For T = 2, T = 2.2, T = 2.7 and T = 3, using deltas from third order: T = 2.1 and T = 2:

For T = 2.1, T = 2.7, T = 3 and T = 3.4 using deltas from third order: T = 2.2 and T = 2.1:

### Divided Differences Table

We can now plug these values into the table like so:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| T | Z | Second Order (b2) | Third Order (b3) | Fourth Order (b4) | Fifth Order (b5) |
| **2** | **6** | **17.52** | **37.6** | **12** | **0** |
| **2.1** | **7.752** | **25.04** | **46** | **12** | **0** |
| **2.2** | **10.256** | **52.64** | **56.8** | **12** |  |
| **2.7** | **36.576** | **98.08** | **71.2** |  |  |
| **3** | **66** | **147.92** |  |  |  |
| **3.4** | **125.168** |  |  |  |  |

### The Polynomial

Now we attempt to construct the Newton Interpolating Polynomial using the following formula:

Where:

– is the value of the polynomial at time

– are the coefficients from the divided differences table

- are the T values of our data points

We gather our coefficients:

We stop on the third order since our T = 2.5 falls just above it.

Now we substitute the values:

Solve for :

By running the code in Listing [1], we get the following results:

A screen shot of a graph

Description automatically generated

The consistency of twelves in the 3rd order and the presence of Zero’s in the fourth order suggests that the data was likely generated by a third-order polynomial (cubic polynomial). In Newton’s method, if the *n*-th order divided differences are zero, it implies that the polynomial of best fit is of order *n – 1*. Since the fourth order divided differences are zero, the polynomial used to generate the data is of order 3, i.e., cubic polynomial.

## Using Lagrange Interpolating Polynomial

To solve the problem using third-order Lagrange interpolating polynomial, we use the data points given.

### General Form

The general form of the Lagrange interpolating polynomial is given as:

Where are the Lagrange Basis Polynomials defined as:

### Basis Polynomials

For a third-order polynomial n = 3, and the basis polynomials for I = 0, 1, 2, 3 are:

With these values, coupled with our values:

### Compute Using Interpolating Polynomial

### MATLAB Implementation

Using the code in Listing [2] to find the Lagrange Coefficient, and then the code in Listing [3] Evaluate using Lagrange’s method, we get the following results

A white paper with black numbers and lines

Description automatically generated

The result shows that the Lagrange method for polynomial interpolation is faster to reach a value than the Newtonian method.

# Problem 2

The following data define the sea-level concentration of dissolved oxygen for fresh water as a function of temperature:

A close up of numbers

Description automatically generated

Use MATLAB to fit the data with:

1. Piecewise linear interpolation
2. A fifth-order polynomial, and
3. A Spline

Display the results graphically and use each approach to estimate *o*(27). Note that the exact result is 7.986.

## Plotting Using Piecewise Linear Interpolation

Using the code in Listing [4], we get:

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Description automatically generated

The task at hand requires us to analyze the concentration of dissolved oxygen in fresh water at sea level as it varies with temperature. The dataset provided encapsulates distinct temperature readings alongside their corresponding oxygen concentrations. The ultimate goal is to utilize piecewise linear interpolation to estimate the concentration of dissolved oxygen at a temperature of 27°C, for which an exact value of 7.986 mg/L has been established for comparison purposes. The script used in Listing [2] executes with the following structure:

### Explanation of Code

The script starts by initializing two arrays representing temperature in degrees Celsius and the corresponding oxygen concentration in mg/L. It also defines the exact value of oxygen concentration at 27°C for later comparison. A piecewise linear interpolation function is then created using the `interp1` function provided by Octave, which is capable of interpolating within the range of given data points.

The script proceeds to estimate the oxygen concentration at 27°C by invoking the interpolation function with 27 as the input. It calculates the error by subtracting the interpolated value from the exact value. Both the estimated oxygen concentration and the calculated error are printed to the console for the user to see.

For visualization, the script generates a plot that includes the given data points marked as red circles, the piecewise linear interpolation as a blue line across a densely populated range of temperature values, and the estimated value at 27°C displayed as a green square. It also plots the exact value at 27°C as a magenta star for reference.

To illustrate the accuracy of the interpolation, the script draws a dashed black line between the estimated and exact oxygen concentration values at 27°C, which visually represents the error. The plot is then finalized with appropriate labels, title, legend, and a grid, and the figure hold is released, allowing for new plots to be created independently of this one.

### Reasoning

In approaching the task of fitting the given temperature and dissolved oxygen data with an interpolation model, I opted for a piecewise linear interpolation method, executed in MATLAB. This choice was influenced by several key factors related to both the nature of the data and the requirements of the problem at hand.

Firstly, the dataset, which represents the concentration of dissolved oxygen at varying temperatures, suggests a non-linear relationship. However, without an underlying theoretical model or a more extensive dataset to justify a higher-order polynomial or a non-parametric fit, a piecewise linear approach offers a balance between simplicity and flexibility. It allows for a model that can adapt to local changes in the trend without overfitting, which is a common risk with high-order polynomials, especially when extrapolating beyond the range of the data.

Piecewise linear interpolation connects each pair of adjacent data points with a straight line, making no assumptions about the data between these points other than continuity. This is particularly suitable for our dataset because the exact relationship between temperature and oxygen concentration in water bodies can be complex, influenced by various environmental factors that are not captured in a simple table of values.

Additionally, a piecewise linear model is computationally efficient and straightforward to implement using MATLAB's built-in interp1 function. This efficiency is a significant consideration when working with potentially large datasets in real-world applications. By leveraging MATLAB's optimized functions, we can achieve accurate interpolations rapidly, which is crucial for timely analysis.

For the specific task of estimating the dissolved oxygen concentration at 27°C, the piecewise linear model provides a direct method to infer the value based on the two data points that bracket this temperature. This interpolation assumes that changes between the known data points occur linearly, which is a reasonable assumption in the absence of more detailed data. In the context of the assignment, displaying the results graphically is not just about fulfilling a requirement; it's about providing a visual confirmation of the model's validity. The plot generated by the script clearly illustrates how the piecewise linear segments pass through the data points and how the model behaves at the interpolation point of interest (27°C).

Finally, the exact result of 7.986 mg/L at 27°C is used as a benchmark to evaluate the accuracy of our interpolation. By comparing the interpolated value to this exact result, we can assess the performance of our piecewise linear model. This comparison is a common practice in scientific computing, where validating models against known or established results is essential for credibility.

## Using a Fifth-Grade Polynomial

Solving the problem using a fifth-order polynomial involves fitting a polynomial of the form:

to the given data points, and then using this polynomial to estimate the dissolved oxygen concentration at 27°C. We'll use MATLAB's ***polyfit*** function for fitting and ***polyval*** for evaluation.

Using the code in Listing [5]

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### Explanation of the Code

In the given code, we establish a foundation for numerical analysis by defining two vectors, T and O, which hold temperature data and corresponding oxygen concentrations. The MATLAB function polyfit is employed to fit a fifth-order polynomial to the data points. This degree is specifically chosen because it allows the polynomial to pass exactly through all six data points provided.

We then use polyval, a function that evaluates a polynomial for a given set of coefficients at a specified point—in this case, at 27°C—to estimate the oxygen concentration. The estimated value is displayed in the MATLAB command window with a precision of three decimal places.

For visualization purposes, a dense range of temperature points called T\_dense is generated using linspace, which allows for plotting a smooth curve representing the polynomial. Corresponding oxygen concentrations over this range, O\_dense, are calculated using the polynomial coefficients obtained from polyfit.

The plotting section of the code begins with the creation of a new figure window. The original data points are plotted as distinct red circles, and the piecewise linear fit and the estimated oxygen concentration at 27°C are overlaid on the same graph, the latter represented as a green square. The hold on command is used to retain the current plot while adding new elements.

The plot is then annotated with labels for both axes, a title, and a legend to provide context and enhance readability. A grid is also enabled, which aids in visual analysis of the data points relative to the polynomial fit.

Lastly, the code prints out the exact oxygen concentration at 27°C, serving as a benchmark for the accuracy of the polynomial interpolation. This allows for a direct comparison between the estimated and known values, highlighting the precision of the polynomial model. On the plot, the error is visualized by a magenta pentagon marking the exact value and a black dashed line connecting this point to the estimated value, clearly illustrating the difference between the two.

### Reasoning

In the provided Octave code snippet, the purpose is to fit a given dataset with a piecewise linear model and to evaluate the model's accuracy at a specific point. The dataset consists of temperature values and their corresponding sea-level concentrations of dissolved oxygen.

The `interp1` function is employed for its ability to perform linear interpolation between data points, which is a straightforward and computationally efficient method for estimating values within the range of known data points. This function is particularly suitable for cases where data varies linearly or near-linearly between known values, as it might be expected with physical measurements like temperature and concentration.

The estimated oxygen concentration at 27°C is computed directly from the interpolation function. This step is critical to provide an immediate application of the interpolation, assessing how the model performs at a temperature value not explicitly included in the original dataset.

Printing the estimated concentration alongside the known exact concentration serves a dual purpose: it provides an immediate visual comparison for the user and gives a tangible measure of the interpolation's accuracy by presenting the error magnitude.

The decision to create a dense temperature range and plot the interpolated values is driven by the need to visualize the linear interpolation across the entire temperature range, ensuring that the model's behavior is clear and can be easily compared to the original data points.

Finally, highlighting the estimated value at 27°C and drawing an error line to the exact value is a deliberate choice to emphasize the model's performance at this specific temperature, illustrating the practical utility of the interpolation and the significance of the error in a real-world measurement scenario.

## Using Splines

To interpolate the given data using splines in MATLAB, we use the spline function shown in the code in Listing [6]. Splines provide a way to interpolate the data points with a piecewise polynomial function that has a specified degree, usually cubic. Splines are particularly useful because they can provide a smoother approximation to the data compared to high-degree polynomials, which can oscillate wildly.

The process for using spline interpolation will be to fit the spline to the data, evaluate the spline at the temperature of interest (27°C), and then plot both the data and the spline for visual analysis.

A screenshot of a computer

Description automatically generated

### Explanation of the Code

% Given data points

T = [0, 8, 16, 24, 32, 40]; % Temperature in degrees Celsius

O = [14.621, 11.843, 9.870, 8.418, 7.305, 6.413]; % Oxygen concentration in mg/L

Here, we start by defining two vectors, T for temperature and O for dissolved oxygen concentration, based on the data provided. This sets the foundation for our interpolation task.

% Fit a spline to the data

spline\_fit = spline(T, O);

Using the spline function, MATLAB computes a cubic spline fit. A cubic spline is a series of piecewise cubic polynomials between each pair of points, with constraints to ensure that the polynomials join smoothly. The result is a spline fit that minimizes sudden changes in the derivative, providing a smooth curve that naturally follows the data's pattern.

% Estimate oxygen concentration at 27 degrees Celsius using the spline

O\_at\_27\_spline = ppval(spline\_fit, 27);

With ppval, we evaluate the spline at 27°C to estimate the oxygen concentration. The spline fit is a model of the underlying trend, and evaluating it at a specific point gives us an interpolated value based on that model.

% Display the estimated oxygen concentration from the spline

fprintf('The estimated oxygen concentration at 27°C using spline interpolation is %.3f mg/L\n', O\_at\_27\_spline);

We use fprintf to print out the interpolated value, giving us a sense of how well the spline has captured the trend at 27°C. We format the output to three decimal places for precision.

% Plot the data and the spline interpolation

T\_dense = linspace(min(T), max(T), 1000); % A dense range for a smooth plot

O\_dense\_spline = ppval(spline\_fit, T\_dense);

To prepare for plotting, we generate a dense array of temperature values (T\_dense) that span the full range of our data. This allows us to plot a smooth curve for the spline. We then calculate the corresponding oxygen concentrations over this dense temperature range (O\_dense\_spline) to visualize the spline's behavior across the entire temperature domain.

figure; % Create a new figure

plot(T, O, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8); % Plot the data points

hold on; % Hold the figure for the next plot

plot(T\_dense, O\_dense\_spline, 'b-'); % Plot the spline interpolation

plot(27, O\_at\_27\_spline, 'ks', 'MarkerFaceColor', 'g'); % Plot the estimated value at 27°C

Here, we create a new figure window. We plot the original data points as red circles to make them stand out, and then we plot the spline curve as a blue line. We also plot a green square to indicate the interpolated value at 27°C.

xlabel('Temperature (°C)');

ylabel('Oxygen Concentration (mg/L)');

title('Spline Interpolation of Oxygen Concentration');

legend('Data Points', 'Spline Fit', 'Estimated Value at 27°C');

grid on; % Add a grid for better readability

We label the axes and add a title, legend, and grid to the plot to enhance readability and provide context to the viewer.

% Print the exact value for comparison

fprintf('The exact oxygen concentration at 27°C is 7.986 mg/L\n');

Finally, we print out the exact value of the oxygen concentration at 27°C for comparison purposes.

### Reasoning

In this portion of the code, we're introduced to the process of fitting a cubic spline to the given temperature and oxygen concentration data. Cubic splines are preferable for creating a smooth curve that can accurately interpolate values between known data points. The MATLAB function `spline` is utilized to construct the spline, which creates a series of piecewise cubic polynomials that pass through each data point with continuous first and second derivatives, ensuring a smooth transition between segments.

The function `ppval` is then employed to evaluate the spline at 27 degrees Celsius. This is a critical step, as it allows us to estimate the oxygen concentration at a temperature that is not explicitly included in the given data set, demonstrating the spline's predictive capabilities.

The estimated oxygen concentration obtained from the spline interpolation is printed to provide immediate feedback on the spline's performance at 27°C. The output is formatted to three decimal places to reflect the precision of the estimation.

To visualize the spline's fit, a dense array of temperature values is generated, covering the full range of the original dataset. This dense array is used to create a smooth plot of the spline interpolation, providing a visual representation of how the spline model fits the entire range of data.

A new figure window is created for plotting, where the original data points are marked as red circles for distinction. The spline interpolation is then plotted as a smooth blue line to show the continuity and smoothness of the spline fit. A green square marker is added to highlight the estimated value at 27°C on the graph.

Finally, the code concludes with labels, title, and legend being added to the plot, as well as a grid to aid in readability. The exact oxygen concentration at 27°C is also printed out, serving as a benchmark to evaluate the accuracy of the spline interpolation, and offering a concrete point of comparison for the spline's estimated value.

# Problem 3

The following model, based on a simplification of the Arrhenius equation, is frequently used in environmental engineering to parameterize the effect of temperature, T (°C), on pollutant decay rates, k (per day),

Where the parameters = the decay rate at 20 °C, and = the dimensionless temperature coefficient. The following data are collected in the laboratory:

A red and white rectangular object with numbers

Description automatically generated

1. Use a transformation to linearize this equation and then employ linear regression to estimate and
2. Employ nonlinear regression to estimate the same parameters. For both a) and b) employ the equation to predict the reaction at *T* = 17°C

## Linearizing the Equation by Transformation

The Arrhenius-type equation provided can be linearized by taking the natural logarithm of both sides, which gives us a linear relationship that can be solved using linear regression:

Given:

Take the natural logarithm of both sides:

This equation is now in the form of *y = mx + b*, where:

*y =*

*m =*

x =

b =

Now we transform the data:

Subtracting 20 from each value:

## Employ Linear Regression

To find the best-fitting line , we calculate the slope m and the intercept b using these formulae for linear regression:

Where is the number of data points which is 5.

We calculate m and b using the transformed data T’ and K’. First, we compute:

Now we substitute these values to calculate both m and b:

Now that we have m and b, can go back to calculate and by exponentiating b to get and exponentiating m to get :

Thus, the estimated decay rate at 20 °C ( is approximately 0.357 per day, and the estimated dimensionless temperature coefficient is approximately 1.067. These estimates are used to model the decay rate as a function of temperature in the form:

Using the code in Listing [7], we can plot the decay rate as a function of temperature:

A screenshot of a computer

Description automatically generated

# Problem 4

The force on a sailboat mast can be represented by the following function:

Where:

= the elevation above the deck

= the heigh of the mast

The total force exerted on the mast can be determined by integrating this function over the height of the mast:

The line of action can also be determined by integration:

1. Use the composite trapezoidal rule to compute and for the case where
2. Repeat *a),* but use the composite Simpson’s rule.

## Using Composite Trapezoidal Rule

To compute and perform subinterval calculations:

This divides the mast into 6 equal parts, each 5 meters long. This provides us with values at which we’ll evaluate the function: 0, 5, 10, 15, 20, 25 and 30 meters.

Evaluating at these values of

Using the following composite trapezoidal rule formula:

Where,

Is the width of each subinterval, and for = 1, 2, …, n – 1

We start using the composite trapezoidal formula for :

Now, we go onto to find using the values found with :

Bracket part:

Adding to fraction:

Using the code in Listing [8], we can see how this function behaves:

A screenshot of a computer

Description automatically generated

We can clearly see that the result of the Trapezoidal formula using the parameters for the exercise converge onto:

## Using Simpson’s rule

To apply Simpson’s rule we must divide the range of integration into intervals, where must be an even number. In this case, and , which is already even. Each interval will have a width of .

First, calculate , the width of each interval:

Next, calculate -values at which we will evaluate the function . With , we will have values.

For each , calculate , since we already have which is:

We now use the formula for Simpson’s rules:

We know from the previous exercise that:

And .

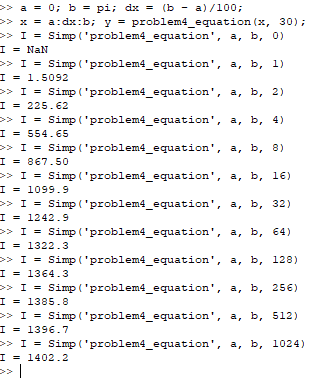
Now, to find :

Where :

Note how we performed the calculations in the order as they appear in the formula for easier plug and play.

Now that we have the values for each term, taking the fact that and :

Using the code in Listing [9], we can see how using Simpson’s rule also converges slowly onto 1402:



# Problem 5

In the investigation of a homicide or accidental death, it is often important to estimate the time of death. From the experimental observations, it is known that the surface temperature of an object changes at a rate proportional to the difference between the temperature of the object and that of the surrounding environment or ambient temperature. This is known as Newton’s law of cooling. This, if is the temperature of the object at time , and is the constant ambient temperature:

where is a constant of proportionality. Suppose that at time a corpse is discovered, and its temperature is measured to be . We assume that at the time of death, the body temperature was at the normal value of 37°C. Suppose that the temperature of the corpse when it was discovered was 29.5°C, and that two hours later, it is 23.5°C. The ambient temperature is 20°C.

1. Determine and the time of death.
2. Solve the ODE numerically and plot the results

## Determining Time of Death

To determine the time of death, we must first find the constant of proportionality basing ourselves from Newton’s law of cooling. We have the following data in our hands:

Newton’s law of cooling is given by the differential equation:

We can solve this differential equation by first attempting to find

Integrating both sides, we get:

### Initial Condition

Where is the constant of integration. We can solve for using the initial condition at

Now we have

And finally, we remove the natural log:

### Second Condition

Now that we have the proper exponential equation, we solve for using the second condition. Taking into account that for the second condition:

Substitute these values inside the equation:

And then solve for :

### Solving Time of Death

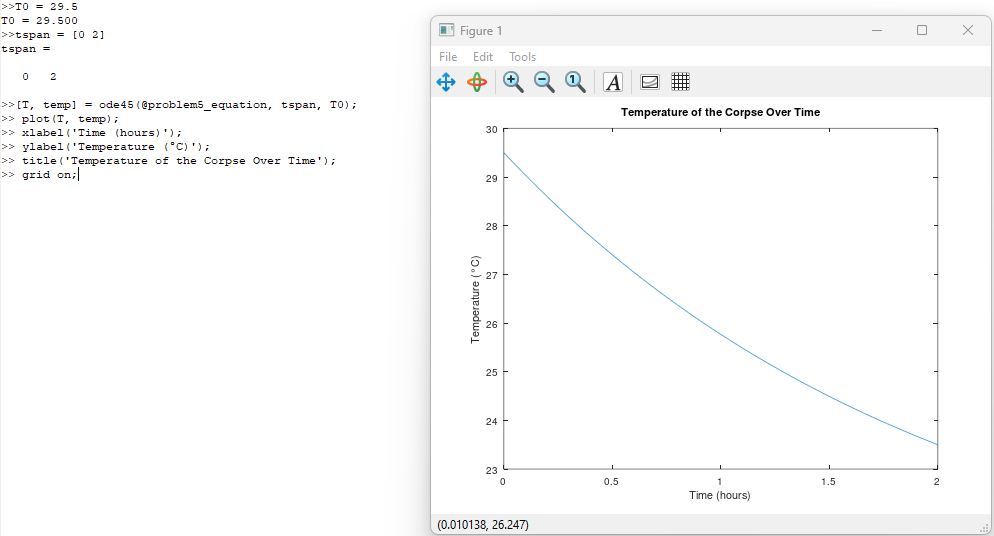
Using this value for , we can now solve the time of death. We go back to our exponential equation:

And substitute the values needed:

And then we solve for :

## Solving it Numerically with MATLAB

Defining our equation as shown in Listing [10], we can use the commands in Listing [11] to plot the results using MATLAB:



# Appendix

## 1 – Script for Newton Interpolation

function [b, yint] = Newtint2(x, y, xx)

% Newtint2(x, y, xx):

% Newton interpolation. Uses an (n-1)-order Newton

% interpolating polynomial based in n data points (x, y)

% to evaluate the interpolated values of the dependent variable (yint)

% at selected locations, xx.

% input:

% x = independent variable

% y = dependent variable

% xx = values of independent variable at which interpolation is calculated

% output:

% yint = interpolated values of dependent variable

%

% compute the finite divided differences in the form of a difference table

n = length(x);

% check the table size

if length(y) ~= n, error('x and y must be same length'); end

b = zeros(n,n);

% assign dependent variables to the first column of b.

b(:,1) = y(:); % the (:) ensures that y is a column vector.

for j = 2:n

for i = 1:n-j+1

b(i,j) = (b(i+1,j-1) - b(i,j-1))/(x(i+j-1) - x(i));

end

end

% use the finite divided differences to interpolate

yint = zeros(size(xx));

for k = 1:length(xx)

xt = 1;

yint(k) = b(1,1);

for j = 1:n-1

xt = xt\*(xx(k) - x(j));

yint(k) = yint(k) + b(1,j+1) \* xt;

end

end

% Plotting part begins here

% Define a dense range of points for plotting the polynomial

x\_dense = linspace(min(x), max(x), 100);

y\_dense = zeros(size(x\_dense));

% Calculate the interpolated values for the dense range

for k = 1:length(x\_dense)

xt = 1;

y\_dense(k) = b(1,1);

for j = 1:n-1

xt = xt \* (x\_dense(k) - x(j));

y\_dense(k) = y\_dense(k) + b(1,j+1) \* xt;

end

end

% Plot the original data points

plot(x, y, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8);

hold on; % Hold on to plot the polynomial on the same figure

% Plot the interpolating polynomial

plot(x\_dense, y\_dense, 'b-', 'LineWidth', 2);

xlabel('Independent variable x');

ylabel('Dependent variable y');

title('Newton Interpolating Polynomial');

legend('Data Points', 'Interpolating Polynomial');

hold off; % Release the plot hold

end

## 2 – Lagrange Coefficient Script

function c = Lagrange\_coef(x, y)

n = length(x)

for k = 1 : n

d(k) = 1;

for i = 1 : n

if i ~= k

d(k) = d(k)\*(x(k) - x(i));

endif

c(k) = y(k)/d(k);

endfor

endfor

end

## 3 – Lagrange Eval Script

function p = Lagrange\_Eval(t, x, c)

m = length(x)

for i = 1 : length(t)

p(i) = 0;

for j = 1 : m

N(j) = 1;

for k = 1 : m

if (j ~= k)

N(j) = N(j) \* (t(i) - x(k));

endif

endfor

p(i) = p(i) + N(j) \* c(j);

endfor

endfor

end

## 4 – Piecewise Interpolation Script

% Given data points

T = [0, 8, 16, 24, 32, 40]; % Temperature in degrees Celsius

O = [14.621, 11.843, 9.870, 8.418, 7.305, 6.413]; % Oxygen concentration in mg/L

% Fit piecewise linear interpolation

piecewiseLinear = @(T\_query) interp1(T, O, T\_query, 'linear');

% Estimate oxygen concentration at 27 degrees Celsius

O\_at\_27 = piecewiseLinear(27);

% Display the estimated oxygen concentration

fprintf('The estimated oxygen concentration at 27°C is %.3f mg/L\n', O\_at\_27);

% Calculate the error at 27 degrees Celsius

exact\_O\_at\_27 = 7.986; % Exact oxygen concentration at 27°C

error\_at\_27 = exact\_O\_at\_27 - O\_at\_27; % Error calculation

fprintf('The error in the estimated value at 27°C is %.3f mg/L\n', error\_at\_27);

% Plot the data and the piecewise linear interpolation

T\_dense = linspace(min(T), max(T), 1000); % A dense range for a smooth plot

O\_dense = piecewiseLinear(T\_dense);

figure; % Create a new figure

plot(T, O, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8); % Plot the data points

hold on; % Hold the figure for the next plot

plot(T\_dense, O\_dense, 'b-'); % Plot the piecewise linear interpolation

plot(27, O\_at\_27, 'ks', 'MarkerFaceColor', 'g'); % Plot the estimated value at 27°C

% Plot the exact value and the error line

plot(27, exact\_O\_at\_27, 'p', 'MarkerFaceColor', 'm', 'MarkerSize', 8); % Plot the exact value

plot([27, 27], [O\_at\_27, exact\_O\_at\_27], 'k--'); % Plot a line representing the error

% Update the legend to include the exact value and error line

legend('Data Points', 'Piecewise Linear Fit', 'Estimated Value at 27°C', 'Exact Value', 'Error at 27°C');

% Finalize the plot with labels, title, and grid

xlabel('Temperature (°C)');

ylabel('Oxygen Concentration (mg/L)');

title('Piecewise Linear Interpolation of Oxygen Concentration');

grid on; % Add a grid for better readability

hold off; % Release the plot hold

## 5 – Fifth-Grade Polynomial Script

% Given data points

T = [0, 8, 16, 24, 32, 40]; % Temperature in degrees Celsius

O = [14.621, 11.843, 9.870, 8.418, 7.305, 6.413]; % Oxygen concentration in mg/L

% Fit a fifth-order polynomial to the data

p = polyfit(T, O, 5);

% Estimate oxygen concentration at 27 degrees Celsius

O\_at\_27 = polyval(p, 27);

% Display the estimated oxygen concentration

fprintf('The estimated oxygen concentration at 27°C using a fifth-order polynomial is %.3f mg/L\n', O\_at\_27);

% Plot the data and the polynomial interpolation

T\_dense = linspace(min(T), max(T), 1000); % A dense range for a smooth plot

O\_dense = polyval(p, T\_dense);

figure; % Create a new figure

plot(T, O, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8); % Plot the data points

hold on; % Hold the figure for the next plot

plot(T\_dense, O\_dense, 'b-'); % Plot the polynomial interpolation

plot(27, O\_at\_27, 'ks', 'MarkerFaceColor', 'g'); % Plot the estimated value at 27°C

xlabel('Temperature (°C)');

ylabel('Oxygen Concentration (mg/L)');

title('Fifth-Order Polynomial Interpolation of Oxygen Concentration');

legend('Data Points', 'Polynomial Fit', 'Estimated Value at 27°C');

grid on; % Add a grid for better readability

% Print the exact value for comparison

fprintf('The exact oxygen concentration at 27°C is 7.986 mg/L\n');

## 6 – Splines Scripts

% Given data points

T = [0, 8, 16, 24, 32, 40]; % Temperature in degrees Celsius

O = [14.621, 11.843, 9.870, 8.418, 7.305, 6.413]; % Oxygen concentration in mg/L

% Fit a spline to the data

spline\_fit = spline(T, O);

% Estimate oxygen concentration at 27 degrees Celsius using the spline

O\_at\_27\_spline = ppval(spline\_fit, 27);

% Display the estimated oxygen concentration from the spline

fprintf('The estimated oxygen concentration at 27°C using spline interpolation is %.3f mg/L\n', O\_at\_27\_spline);

% Plot the data and the spline interpolation

T\_dense = linspace(min(T), max(T), 1000); % A dense range for a smooth plot

O\_dense\_spline = ppval(spline\_fit, T\_dense);

figure; % Create a new figure

plot(T, O, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8); % Plot the data points

hold on; % Hold the figure for the next plot

plot(T\_dense, O\_dense\_spline, 'b-'); % Plot the spline interpolation

plot(27, O\_at\_27\_spline, 'ks', 'MarkerFaceColor', 'g'); % Plot the estimated value at 27°C

xlabel('Temperature (°C)');

ylabel('Oxygen Concentration (mg/L)');

title('Spline Interpolation of Oxygen Concentration');

legend('Data Points', 'Spline Fit', 'Estimated Value at 27°C');

grid on; % Add a grid for better readability

% Print the exact value for comparison

fprintf('The exact oxygen concentration at 27°C is 7.986 mg/L\n');

## 7 – Decay Rate as a Function of Temperature Script

% Given data points

T = [6, 12, 18, 24, 30]; % Temperature in degrees Celsius

k = [0.15, 0.20, 0.32, 0.45, 0.70]; % Decay rates per day

% Given values for k\_20 and theta

k\_20 = 0.357;

theta = 1.067;

% Create a range of temperatures for plotting

T\_range = linspace(min(T), max(T), 100);

% Calculate the decay rates using the model

k\_model = k\_20 \* (theta .^ (T\_range - 20));

% Plotting

figure; % Create a new figure

plot(T, k, 'o', 'MarkerFaceColor', 'r', 'MarkerSize', 8); % Original data points

hold on; % Hold the figure for the next plot

plot(T\_range, k\_model, 'b-'); % Model

% Adding details to the plot

xlabel('Temperature (°C)');

ylabel('Decay Rate (per day)');

title('Decay Rate as a Function of Temperature');

legend('Original Data', 'Model');

grid on; % Add a grid for better readability

% Display the plot

hold off;

## 8 – Composite Trapezoidal Script

function I = Trap(func, a, b, n)

x = a;

h = (b - a)/n;

S = feval(func, a, n);

for j = 1 : n - 1

x = x + h;

S = S + 2 \* feval(func, x, n);

endfor

S = S + feval(func, b, n);

I = (10)\* ((b - a)\*S / (2\*n));

End

## 9 - Simpson’s Rule

function I = Simp(f, a, b, n)

% integral of f using composite Simpson rule

% n must be even

h = (b - a)/n;

S = feval(f,a, n);

for i = 1 : 2 : n-1

x(i) = a + h\*i;

S = S + 4\*feval(f, x(i), n);

end

for i = 2 : 2 : n-2

x(i) = a + h\*i;

S = S + 2\*feval(f, x(i), n);

end

S = S + feval(f, b, n); I = 10\*(h\*S/3);

End

## 10 – Problem 5 Equation

function dTdt = problem5\_equation(t, T)

K = 0.4993;

Ta = 20;

dTdt = -K \* (T - Ta);

end

## 11 – ODE Solve Commands

% Time range for the solution (assuming you want to simulate for 10 hours)

tspan = [0 10];

% Solve the ODE

[T, temp] = ode45(@temperatureChange, tspan, T0);

% Plotting the results

plot(T, temp);

xlabel('Time (hours)');

ylabel('Temperature (°C)');

title('Temperature of the Corpse Over Time');

grid on;