# Problem 1

The following data are measured precisely:

A close up of numbers

Description automatically generated

1. Use Newton interpolating polynomials to determine z at t = 2.5. Make sure that you order your points to attain the most accurate results. What do your results tell you regarding the order of the polynomials used to generate the data?
2. Use a third-order Lagrange interpolating polynomial to determine y at 2.5.

## Using Newton Interpolating Polynomial

Given our points T and Z, we start computing the first order divided difference:

### Formula

We now construct the Divided Difference table using this formula, starting with the First Order.

For T = 2, to T = 2.1:

For T = 2.1 to T = 2.2:

For T = 2.2 to T = 2.7:

For T = 2.7 to T = 3:

For T = 3 to T = 3.4:

For the second order, we use the values gotten when finding the first order divided difference.

For T = 2 to T = 2.2:

For T = 2.1 to T = 2.7:

For T = 2.2 to T = 3:

For T = 2.7 to T = 3.4:

Now, we continue with the third order using the computer values from the second order.

For T = 2, T = 2.2 and T = 2.7 using deltas from second order: T = 2.1 and T = 2:

For T = 2.1, T = 2.7 and T = 3 using deltas from second order: T = 2.2 and T = 2.1:

For T = 2.2, T = 3 and T = 3.4 using deltas from second order: T = 2.7 and T = 2.2:

Finally, we move to the fourth order, we again use the values from the previous order.

For T = 2, T = 2.2, T = 2.7 and T = 3, using deltas from third order: T = 2.1 and T = 2:

For T = 2.1, T = 2.7, T = 3 and T = 3.4 using deltas from third order: T = 2.2 and T = 2.1:

### Divided Differences Table

We can now plug these values into the table liks so:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| T | Z | Second Order (b2) | Third Order (b3) | Fourth Order (b4) | Fifth Order (b5) |
| **2** | **6** | **17.52** | **37.6** | **12** | **0** |
| **2.1** | **7.752** | **25.04** | **46** | **12** | **0** |
| **2.2** | **10.256** | **52.64** | **56.8** | **12** |  |
| **2.7** | **36.576** | **98.08** | **71.2** |  |  |
| **3** | **66** | **147.92** |  |  |  |
| **3.4** | **125.168** |  |  |  |  |

### The Polynomial

Now we attempt to construct the Newton Interpolating Polynomial using the following formula:

Where:

– is the value of the polynomial at time

– are the coefficients from the divided differences table

- are the T values of our data points

We gather our coefficients:

We stop on the third order since our T = 2.5 falls just above it.

Now we substitute the values:

Solve for :

By running the code in Listing [1], we get the following results:

A screen shot of a graph

Description automatically generated

The consistency of twelves in the 3rd order and the presence of Zero’s in the fourth order suggests that the data was likely generated by a third-order polynomial (cubic polynomial). In Newton’s method, if the *n*-th order divided differences are zero, it implies that the polynomial of best fit is of order *n – 1*. Since the fourth order divided differences are zero, the polynomial used to generate the data is of order 3, i.e., cubic polynomial.